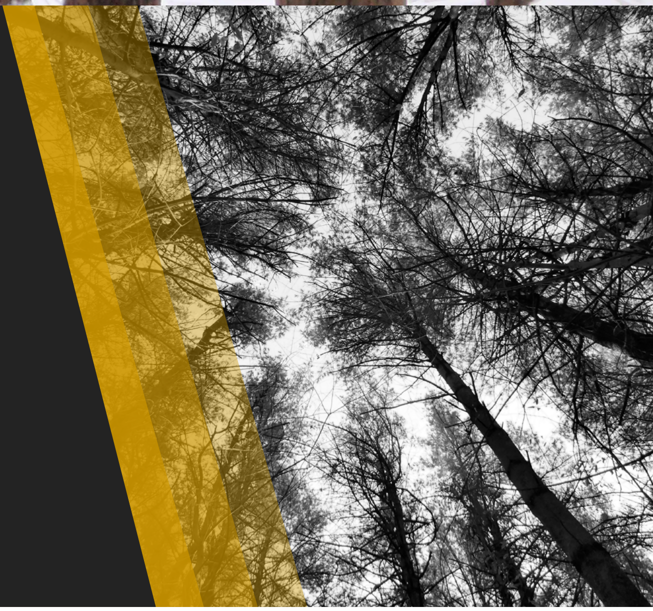
Total and merchantable volume equations for Ontario white spruce and white pine plantations



Technical Report TR-59

Science and Research Branch

Ministry of Natural Resources and Forestry

Ontario

Science and Research Technical Report TR-59

Total and merchantable volume equations for Ontario white spruce and white pine plantations

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2023

Science and Research Branch

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Cite this report as:

Sharma, M. 2023. Total and merchantable volume equations for Ontario white spruce and white pine plantations. Ontario Ministry of Natural Resources and Forestry, Science and Research Branch, Peterborough, ON. Science and Research Technical Report TR-59. 13 p.

Abstract

Accurate tree volume estimates are fundamental to sustainable forest management. Total inside and outside bark and merchantable volume equations were developed for white spruce (*Picea glauca* (Moench.) Voss) and white pine (*Pinus strobus* L.) plantations using data collected from 400 white spruce and white pine trees (200/species) from 80 sites (40/species) in Ontario, Canada. The final volume equations were selected based on fit statistics, predictive accuracy, and logical consistency. Both volume equations were fit using a nonlinear mixed effect modelling approach and a power variance function was used to address heteroscedasticity.

For white pine plantations, a dimensionally compatible volume equation had logically consistent parameter estimates for all volumes, including a positive intercept for total volume and a negative intercept for merchantable volume. Therefore, the dimensionally compatible volume equation was selected for estimating volumes in white pine plantations. For white spruce plantations, however, the combined variable volume equation had logically consistent parameters for total volume and the dimensionally compatible volume equation was logically consistent for merchantable volume. Therefore, the combined variable and dimensionally compatible volume equations were used for estimating total and merchantable volume in white spruce plantations, respectively.

Résumé

Équations relatives au volume total et au volume marchand de plantations d'épinette blanche et de pin blanc de l'Ontario

Des estimations précises du volume des arbres sont essentielles à la gestion durable des forêts. Des équations relatives au volume marchand (avec et sans écorce) ont donc été formulées, notamment pour l'épinette blanche *(Picea glauca* [Moench] Voss) et le pin blanc *(Pinus strobus* L.) à partir de données recueillies sur 400 épinettes blanches et pins blancs (200 par espèce) provenant de 80 sites (40 par espèce) en Ontario, au Canada. Les équations de volume finales ont été sélectionnées de façon à assurer la qualité de l'ajustement sur le plan des statistiques, de l'exactitude prédictive et de la cohérence logique. Les deux équations de volume ont été ajustées à l'aide d'une approche de modélisation non linéaire à effets mixtes. De plus, une fonction de variance de puissance a été utilisée pour traiter l'hétéroscédasticité.

Pour les plantations de pin blanc, une équation modifiée à partir de dimensions compatibles présentait des paramètres logiquement cohérents pour tous les volumes, y compris une interception positive pour le volume total et une interception négative pour le volume marchand. Aussi a-t-on choisi l'équation modifiée à partir de dimensions compatibles pour estimer le volume dans ces plantations. Cependant, pour les plantations d'épinette blanche, l'équation du volume reposant sur des variables combinées présentait des paramètres logiquement cohérents pour le volume total, alors que l'équation modifiée à partir de dimensions compatibles était logiquement cohérente pour le volume marchand. Par conséquent, les deux types d'équation, reposant sur des variables combinées et modifiée à

partir de dimensions compatibles, ont été utilisés pour estimer respectivement le volume total et le volume marchand dans les plantations d’épinette blanche.

Acknowledgements

This study was supported by the Forestry Futures Trust Knowledge Transfer and Tool Development (KTTD) program and the Ontario Ministry of Natural Resources and Forestry. The author is grateful to Chris Stratton and Todd Little, Biodiversity and Monitoring Section, MNRF, for coordinating data collection work and producing sample location map, respectively.

Contents

Introduction

Forest managers develop management plans based on sustainable wood supply, which is calculated from tree (stem) volume. These estimates are therefore necessary to support forest management. Tree volume is calculated using taper or volume equations, which require diameter at breast height (DBH) and total tree height measurements. Taper equations are more flexible than volume equations as they can be used to estimate volume of any stem section and provide information about tree shape (Sharma 2019). Volume equations, however, are easier to use when estimating total tree volume.

Total volume is calculated with and without bark as desired, usually referred to as outside and inside bark volume, respectively. Merchantable volume to a specified upper height or diameter limit is estimated using separate equations, usually without bark. Estimated volume of all individual trees in a stand can be combined to calculate stand volume and converted into volume per unit area (e.g., volume per ha). Tree biomass is calculated by multiplying tree volume and wood density; the amount of carbon stored by a tree is a function of its biomass (Schlesinger 1991). Therefore, accurate tree volume estimates also support improved estimates of tree biomass and forest carbon stocks.

Tree volume depends on shape, which varies by species, stand origin (planted or natural), and stand density. For a given species and stand density, trees grown in plantations taper more than those grown in natural stands (Sharma 2019, 2020). Similarly, for a given stand origin, trees with more surrounding space taper more (Sharma and Parton 2009). Therefore, for a given DBH, total height, and stand density, a tree in a natural stand may contain more volume than one in a plantation. As a result, stand origin-specific volume equations are needed to calculate continuous wood supply and make informed forest management decisions.

Honer (1967) presented total and merchantable volume equations for most commercial trees grown in natural stands in central and eastern Canada, including white spruce (*Picea glauca* (Moench.) Voss) and white pine (*Pinus strobus* L.). Sharma (2021) derived improved total inside and outside bark (hereafter total volume) and merchantable tree volume equations for these species grown in natural stands. However, total (inside and outside bark) and merchantable volume equations for plantation grown white spruce and white pine were not available. Therefore, the objective of this study was to develop total and merchantable volume equations for white spruce and white pine plantations in Ontario.

Methods

Data

The data was obtained by sampling 40 even-aged pure white spruce and white pine plantations (40 plantations per species) across Ontario, Canada, using a variable size circular temporary plot (TSP) established in each plantation (Figure 1). The minimum plot size was 400 m^2, but was increased, if necessary, to include a minimum of 40 trees per species.

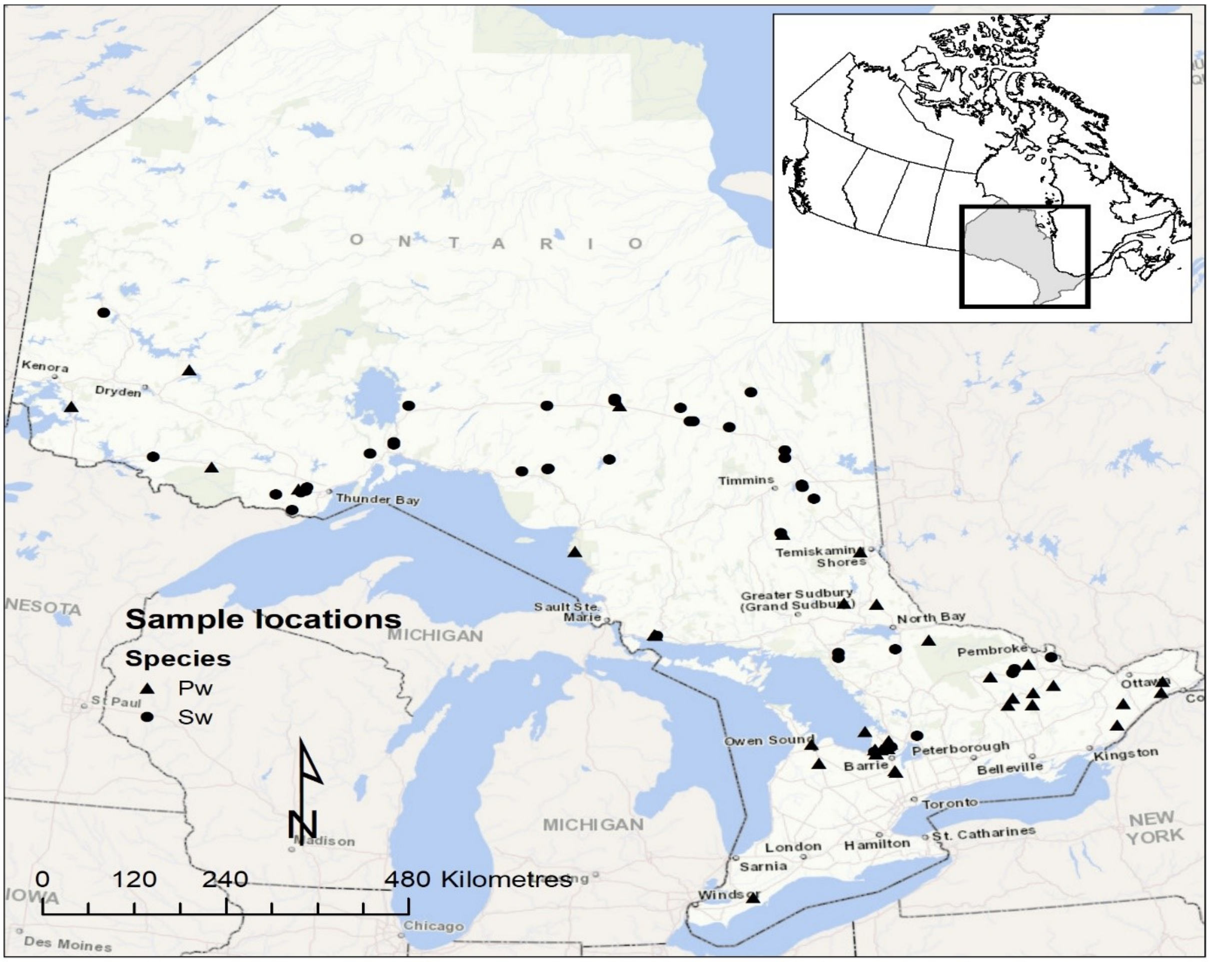

Figure 1. Distribution of white spruce (Sw, circles) and white pine (Pw, triangles), plantation sites sampled across Ontario, Canada.

Sampled trees were measured following Ontario's growth and yield standards (Hayden et al. 1995). Total basal area (BA per ha) and stem density (trees per ha) were calculated for all live trees in each plot. Cumulative basal area was determined by sequentially numbering all live trees of target species growing in the plot. Total cumulative basal area of each plot was divided into 5 classes. One tree from each class was randomly selected for destructive sampling. Only trees confirmed as planted and lacking visible deformities (e.g., major stem injuries or forked, dead, or broken tops) were selected, resulting in 5 sample trees per plot and a total of 200 trees for each species. Summary statistics for all sampled trees and associated stand characteristics are listed in Table 1.

Sampled trees were cut at 3 heights below (0.15, 0.5, 0.9 m) and at breast height (1.3 m) for DBH growth analysis. Trees were also cut at 9 heights above breast height by dividing the remaining height of the tree by 10, resulting in 13 cuts per tree including at breast height. The largest outside and inside bark diameters and those perpendicular to them, all passing through the pith, were measured at stem heights where sections were cut. Mean inside and outside bark diameters were obtained by averaging these diameters at that stem height for each species.

Table 1. Summary statistics for diameter at breast height (DBH), total height, crown ratio, and inside and outside bark and merchantable volumes of sampled white spruce and white pine trees (N=200) in Ontario.

Species/attribute	Mean	Std Dev	Minimum	Maximum
White spruce				
DBH (cm)	27.78	8.84	11.50	55.10
Total height (m)	21.09	4.59	8.60	34.90
Outside bark volume (m^3)	0.7521	0.6154	0.0659	3.7844
Inside bark volume (m^3)	0.6781	0.5591	0.0533	3.5072
Merchantable volume (m^3)	0.6462	0.5430	0.0422	3.4086
White pine				
DBH (cm)	24.83	7.03	10.10	48.80
Total height (m)	19.59	3.08	12.30	26.75
Outside bark volume (m^3)	0.5289	0.3525	0.0517	2.3839
Inside bark volume (m^3)	0.5012	0.3360	0.0488	2.2768
Merchantable volume (m^3)	0.4710	0.3212	0.0347	2.1682

Inside and outside bark volumes of sections between 2 consecutive cuts were calculated using Smalian's formula (Avery and Burkhart 2001). Volumes below the bottom section and above the top section were calculated by assuming a cylinder and cone, respectively. All section volumes calculated using inside bark diameters were combined to obtain total inside bark volume for that tree. Similarly, all section volumes calculated using outside bark diameters were summed to obtain the total outside bark volume for that tree. Merchantable volume was calculated by adding the inside bark section volumes from the stump (30 cm) to 7 cm inside bark diameters without correcting for defect, breakage, or trim. Summary statistics for total and merchantable volumes of white spruce and white pine are listed in Table 1.

Volume equations

While diameter used in volume calculations is usually measured at the base of a cone, tree diameter is measured at breast height (1.3 m). Since some volume (inside or outside bark) is present below breast height, even if DBH is zero total volume should always be more than zero. Similarly, estimated outside bark volume should always be higher than inside bark regardless of tree size. These properties are crucial for a logically consistent tree volume equation. Logically consistent estimates of tree volume can be obtained by incorporating theoretical information with volume equation development, such as the dimensional analysis technique (Sharma and Oderwald 2001) that was considered in this study. Its mathematical expression is:

$$V = \beta D^{\gamma} H^{3-\gamma} + \varepsilon \qquad (1)$$

where, V = total tree volume (inside/outside bark, m^3)

D = diameter at breast height (DBH; m)

H = total tree height (m)

β and γ are parameters to be estimated, and ε is the error term.

Equation (1) assumes V=0 when D=0. Therefore, only volume above breast height can be estimated using equation 1. To estimate total volume, Sharma (2019) added a constant (parameter), α, to the right-hand side of the equation, ensuring that V is the volume accumulated before the tree reaches breast height, i.e.,

$$V = \alpha + \beta D^{\gamma} H^{3-\gamma} + \varepsilon \qquad (2)$$

In mathematics, the volume of a circular base solid with base diameter D and height H is expressed as:

$$V = \beta D^2 \mathrm{H} \qquad (3)$$

where, β=π/4, π/8, π/12, and π/16 for a cylinder, paraboloid, cone, and neiloid, respectively. If the shape of white spruce and white pine trees is one of these solids, tree volume can be expressed as:

$$V = \alpha + \beta D^2 \mathrm{H} + \varepsilon \qquad (4)$$

In this case, α is the volume accumulated by a tree before reaching breast height. Since tree shape is unknown, β is also unknown and must be estimated using the data. This equation, known as a combined variable volume equation (Spurr 1952), has been used to estimate tree volume.

For logical consistency, the intercept in volume equations (2) and (4) should be positive (α>0) for both outside and inside bark volumes. When estimating merchantable volume, stem diameters less than the merchantable diameter limit are excluded, and the intercept is negative. Usually, it is equal to the total inside bark volume of a tree until it reaches the merchantable limit (7 cm inside bark diameter of bottom section).

Equations (2) and (4) were fit using mixed-effects models in SAS (SAS Institute Inc. 2015) for both species. Random effects were used to account for the two-level hierarchical structure of the data (trees nested in a site; site and tree scale random factors). Parameters and fit statistics (mean square error (MSE), Akaike information criterion (AIC; Akaike 1978), and -2 log likelihood) were estimated and examined to ensure logical consistency. Heteroscedasticity was examined by estimating volumes and calculating residuals (observed − predicted) for each tree per species and plotting residuals against predicted volumes. If present, heteroscedasticity was addressed by specifying a variance function for that model (Pinheiro and Bates 2000).

Power and exponential variance functions were used to address heteroscedasticity and compared using AIC values. The final model was selected based on fit statistics (lowest MSE, AIC, and -2 log likelihood) and logical consistency. To ensure outside bark tree volume was always higher than inside bark volume for all trees, estimated outside bark volume was divided by inside bark volume for each tree and plotted against inside bark volume for each species.

Trees were divided into DBH and height classes and, for each class, bias, absolute bias, and root mean square errors (RMSE) for total (inside and outside bark) and merchantable volume estimates were calculated using the developed equations. Percent bias was also calculated for each DBH and height class. Bias, absolute bias, RMSE, and percent bias were calculated as:

$$Bias = \sum_{i=1}^{n} \frac{(y_i - \hat{y}_i)}{n} \quad (5)$$

$$Absolute\ bias = \sum_{i=1}^{n} \frac{|\, y_i - \hat{y}_i|}{n} \quad (6)$$

$$RMSE = \sqrt{\sum_{i=1}^{n} \frac{(y_i - \hat{y}_i)^2}{n}} \quad (7)$$

$$Percent\ bias = 100 * Bias / y_i \quad (8)$$

where, y_i and $\hat{y}_i$ are the observed and predicted volumes, respectively, and n is the number of trees for a particular DBH and height class.

Results

Equations (2) and (4) were fit to data for both species using NLMIXED procedure in SAS (SAS Institute Inc. 2015) without random effects. For consistency, DBH and total height were expressed in metres and volumes (total and merchantable) in cubic metres. The total volume intercepts were positive but not significant, while the merchantable volume intercept was negative and significant for equation (2) for white spruce. For equation (4), the total volume intercepts were positive and significant but negative and not significant for merchantable volume for this species. Since the intercept should be positive for total volumes and negative for merchantable volume, equation (2) was selected for merchantable volume and equation (4) for total volume for white spruce. These equations were then fit by including random effects associated with all fixed parameters at site and tree scales.

For white pine, the total volume intercept was positive and significant and negative and significant for merchantable volume for equations (2) and (4), respectively. However, the fit statistics were better for equation (2) than (4) for all volumes. Therefore, equation (2) was selected for total and merchantable volumes for white pine. This equation was then fit by including random effects associated with all fixed parameters at site and tree scales.

No significant random effects were associated with the intercepts for either species or equation. Random effects associated with the shape parameter (β) were significant at site scale for all volume equations. The random effect associated with γ in equation (2) was not significant for all volume equations for white pine and merchantable volume for white spruce. The volume equations with random effects at site scale are written as:

White spruce

Total inside and outside bark

$$V_{ij} = \alpha + (\beta + b_{1i}) D_{ij}^{\ 2}\, H_{ij} + \varepsilon_{ij} \quad (9)$$

Merchantable

$$V_{ij} = \alpha + (\beta + b_{1i})D_{ij}{}^{\gamma}\, H_{ij}{}^{3-\gamma} + \varepsilon_{ij} \quad (10)$$

White pine

Total inside and outside bark and merchantable

$$V_{ij} = \alpha + (\beta + b_{1i})D_{ij}{}^{\gamma}\, H_{ij}{}^{3-\gamma} + \varepsilon_{ij} \quad (11)$$

where, V_{ij} is the tree volume of site *i* and tree *j* with DBH and total height as D_{ij} and H_{ij}, respectively. Random effect, b_{1i}, is normally distributed with mean zero and variance σ_b^2 (i.e., $b_{1i} \sim N(0, \sigma_b^2)$). Similarly, ε_{ij}, is normally distributed with mean zero and variance σ_e^2 (i.e., $\varepsilon_{ij} \sim N(0, \sigma_e^2)$). In the presence of heteroscedasticity, σ_e^2 will be multiplied by a variance function. The AIC value was reduced by including b_{1i} for both inside and outside bark and merchantable volumes.

Residuals were calculated for equations (9)–(11) and plotted against predicted volumes, which revealed heteroscedasticity in the data (not shown here) for both species. Therefore, equations (9)–(11) with random effects (b_{1i}s) were fit with power and exponential variance functions for both species. The DBH and HT*DBH^2 terms were the base terms in the variance functions. For both total and merchantable volume equations, the power function with base term DBH resulted in a better fit (smaller MSE and AIC values) for both species and was, therefore, selected as the variance function for all equations. As a result, the error term (ε_{ij}) in equations (9)–(11) was normally distributed with mean zero and variance $\sigma_e^2 D_{ij}{}^{\varphi}$ (i.e., $\varepsilon_{ij} \sim N(0, \sigma_e^2 D_{ij}{}^{\varphi})$) and φ was an estimated parameter.

Estimated parameters and their fit statistics for total and merchantable volumes for white spruce (equations 9–10) and white pine (equation 11) are listed in Table 2. The estimates for intercept (α) were positive and significantly different from zero for both total volumes and negative for merchantable volume for both species. Estimates for other parameters were consistent in sign and magnitude across all volume equations. The weight (power of DBH) was positive for both total and merchantable tree volume equations for both species.

When weight was included, the AIC decreased for both volume equations for both species. Ratio of estimated total volumes plotted against estimated inside bark volumes showed that estimated outside bark volume was always higher than the inside bark volume (figures 2, 3). Estimated total volumes were also plotted against their observed value. For both total and merchantable volumes, estimated values were randomly clustered around the 1:1 line for both species (figures 2, 3).

Accuracy of equations (9)–(11) was evaluated by examining bias, absolute bias, RMSE, and percent bias of total and merchantable volume estimates of white spruce and white pine plantations. Diameter and height of white spruce and white pine trees were grouped into 5 cm and 5 m classes, respectively, and bias, absolute bias, RMSE, and percent bias were calculated for each class. These statistics are listed in Table 3 for white spruce and Table 4 for white pine.

Table 2. Parameter estimates, standard error (Std error), and fit statistics (MSE (σ_e^2), variance of b (σ_b^2), and Akaike's information criterion (AIC)) for inside and outside bark volume (Eq. 9) and merchantable volume (Eq. 10) equations for white spruce and for white pine in Ontario fitted to inside and outside bark and merchantable volume data (Eq. 11) using NLMIXED procedure in SAS. NA=not available.

Species/ parameter	Inside bark volume		Outside bark volume		Merchantable volume	
	Estimate	Std error	Estimate	Std error	Estimate	Std error
White spruce						
α	0.00547	0.00200	0.00584	0.00203	-0.01646	0.00307
β	0.36070	0.00343	0.38080	0.00350	0.12260	0.02317
γ	NA	NA	NA	NA	1.75580	0.04447
φ *	5.00930	0.41680	4.93510	0.42760	4.40590	0.36370
σ_e^2	0.86240	0.53270	0.68610	0.43400	0.31320	0.16890
σ_b^2	0.00008	0.00005	0.00010	0.00005	0.00001	0.00001
AIC	-861.8	NA	-879.3	NA	-883.5	NA
White pine						
α	0.00412	0.00032	0.00478	0.00031	-0.00484	0.00033
β	0.33240	0.04499	0.35040	0.04467	0.31640	0.04549
γ	1.99340	0.03340	1.98120	0.03022	1.99040	0.03428
φ *	4.12420	0.39670	4.54750	0.41530	3.89640	0.39830
σ_e^2	0.27370	0.14450	0.50050	0.27740	0.20340	0.10670
σ_b^2	0.00016	0.00008	0.00017	0.00008	0.00014	0.00007
AIC	-741.0	NA	-730.4	NA	-740.2	NA

* Weight (power of DBH (m)). Note: Before using these estimates, convert DBH to m.

The bias estimates for total and merchantable volume were small for both species (tables 3, 4). For white spruce, percent bias was less than 5.2% for trees with DBH of at least 35 cm for total volume and less than 2.3% for merchantable volume for both DBH and height classes. For white pine, percent bias was less than 2.4% for total and merchantable volumes for both DBH and height classes.

The volume equations therefore accurately estimated total and merchantable volume of white spruce and white pine plantations. The intercept was significant in the regression for both total and merchantable volume equations for both species. The estimated values for the intercept were 0.00547, 0.00584, and -0.01646 m^3 for inside bark, outside bark, and merchantable volumes, respectively for white spruce and 0.0038, 0.0049, and -0.00507 m^3, respectively for white pine.

Since the DBH measurements used to fit equations (9)–(11) were in metres, measurements should be converted before using these equations. These equations also apply to imperial units by converting the intercept values from cubic metres to cubic feet and DBH and total height from metres to feet. The values of the estimates for other parameters remain the same regardless of the unit system.

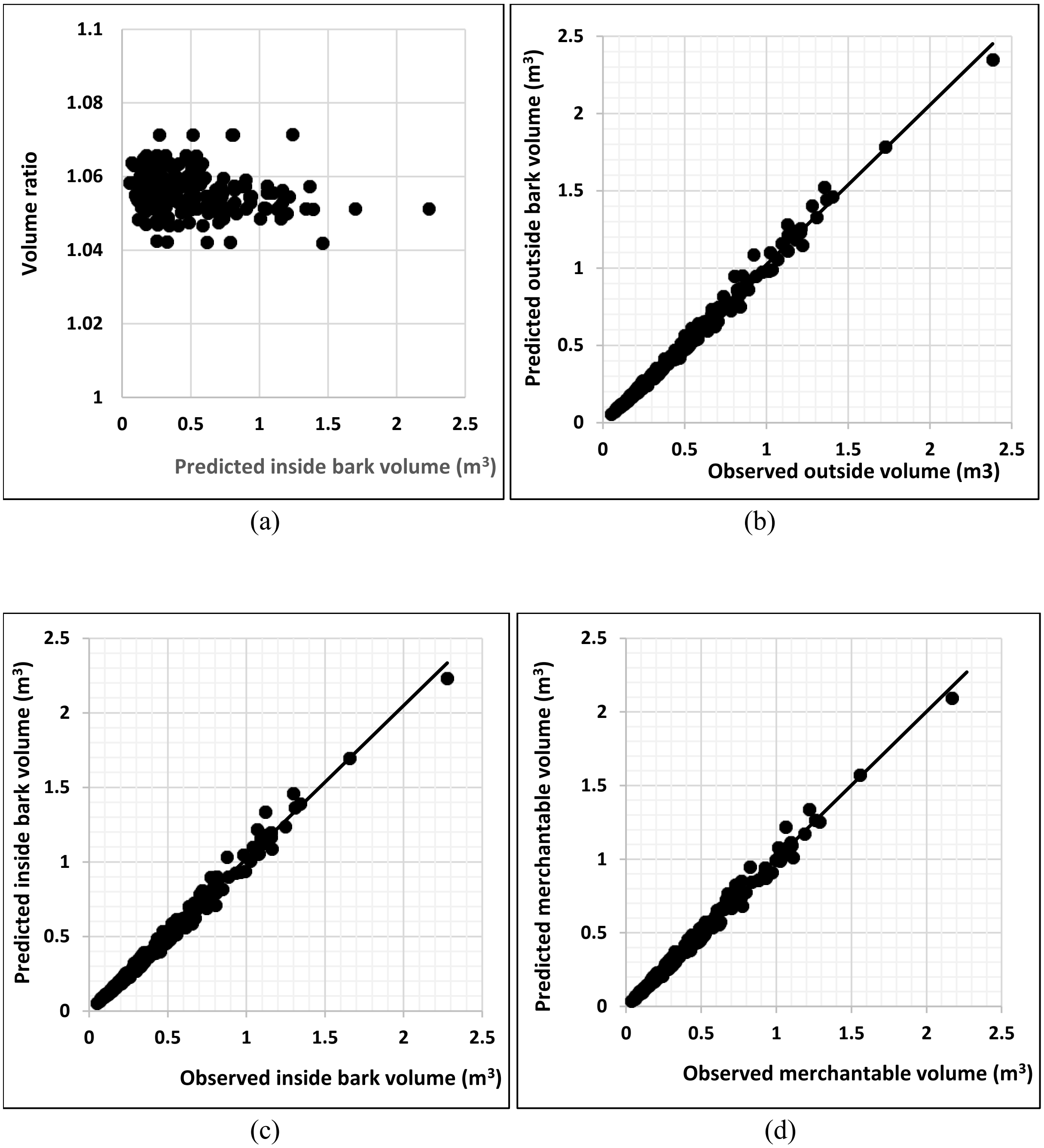

Figure 2. Relationship between (a) the ratio of outside to inside bark volumes estimated using equation (9) and predicted inside bark volumes, and the predicted volumes for (b) outside bark, (c) inside bark, and (d) merchantable volumes against observed counterparts for white spruce plantations in Ontario. The solid line represents the 1:1 line.

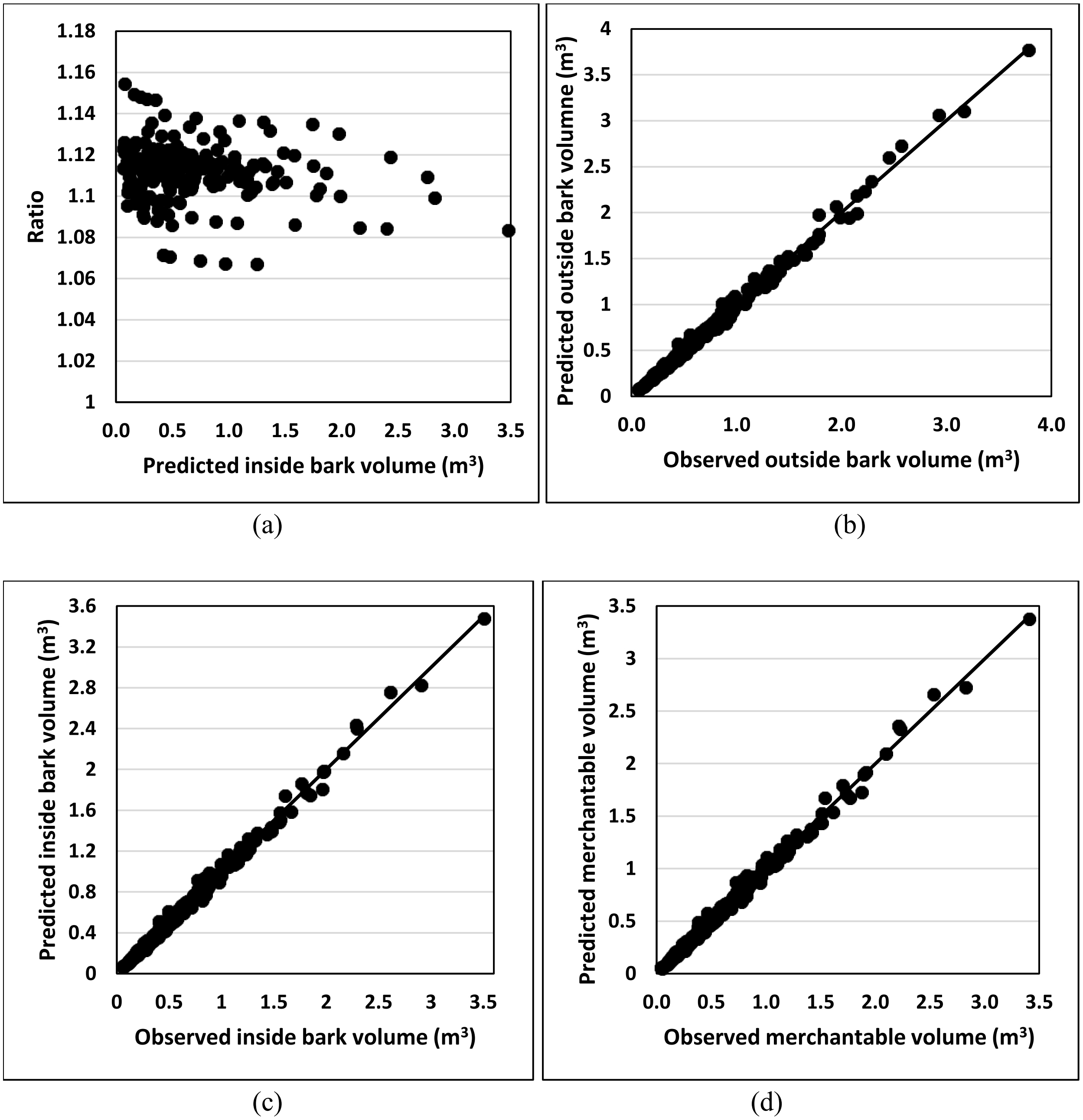

Figure 3. Relationship between (a) the ratio of outside to inside bark volumes estimated using equation (11) and inside bark volumes and predicted and observed volumes for (b) outside bark, (c) inside bark, and (d) merchantable volumes against observed counterparts for white pine plantations in Ontario. The solid line represents the 1:1 line.

Table 3. Bias (observed − predicted), absolute bias, mean square error (RMSE), and percent bias of the residuals from equations for inside and outside bark (Eq. 9) and merchantable volume (Eq. 10) for white spruce plantations in Ontario.

Attribute	Class	N	Bias (m^3)	Absolute bias (m^3)	RMSE (m^3)	Percent bias
Inside bark						
Diameter class (cm)	<15.0	16	-0.00056	0.00575	0.00677	-1.66742
	15.1–20.0	38	0.00285	0.00990	0.01225	0.91587
	20.1–25.0	54	0.00506	0.01688	0.02043	0.93986
	25.1–30.0	50	-0.00110	0.02913	0.03566	-0.70268
	30.1–35.0	22	-0.02307	0.04041	0.05204	-3.35380
	>35.0	19	-0.05478	0.06962	0.09031	-5.06708
Height class (m)	<15.0	11	-0.00227	0.00619	0.00710	-2.35960
	15.1–20.0	103	-0.00042	0.01747	0.02311	-0.06251
	20.1–25.0	77	0.01100	0.03589	0.05250	1.10402
	>25.0	8	-0.03947	0.05215	0.07009	-3.61394
Outside bark						
Diameter class (cm)	<15.0	16	-0.00092	0.00518	0.00645	-1.87754
	15.1–20.0	38	0.00305	0.01001	0.01220	0.96502
	20.1–25.0	54	0.00610	0.01379	0.01690	1.27486
	25.1–30.0	50	-0.00210	0.02666	0.03355	-0.73848
	30.1–35.0	22	-0.02107	0.03288	0.04806	-2.79549
	>35.0	19	-0.05999	0.07256	0.08756	-5.18367
Height class (m)	<15.0	11	-0.00304	0.00630	0.00751	-2.80414
	15.1–20.0	103	0.00008	0.01529	0.02148	0.15171
	20.1–25.0	77	-0.01108	0.03305	0.04885	-0.98287
	>25.0	8	-0.04792	0.05682	0.07299	-3.81770
Merchantable						
Diameter class (cm)	<15.0	16	-0.00003	0.00623	0.00758	-0.92627
	15.1–20.0	38	0.00023	0.00912	0.01185	-0.36340
	20.1–25.0	54	-0.00336	0.01437	0.01839	0.67548
	25.1–30.0	50	0.00214	0.02523	0.03109	0.08422
	30.1–35.0	22	-0.01418	0.03670	0.04695	-2.32697
	>35.0	19	-0.00869	0.05017	0.07171	-1.31888
Height class (m)	<15.0	11	0.00126	0.00578	0.00692	1.35462
	15.1–20.0	103	0.00215	0.01546	0.01991	-0.05481
	20.1–25.0	77	-0.00448	0.02827	0.04172	-0.75763
	>25.0	8	-0.00875	0.05146	0.07387	-1.88905

Table 4. Bias (observed − predicted), absolute bias, mean square error (RMSE), and percent bias of the residuals from equations for total inside and outside bark and merchantable volume (Eq. 11) for white pine plantations in Ontario.

Attribute	Class	N	Bias (m^3)	Absolute bias (m^3)	RMSE (m^3)	Percent bias
Inside bark						
	<15.0	14	-0.00074	0.00554	0.00680	-2.06605
Diameter class (cm)	15.1–20.0	28	0.00493	0.01013	0.01407	2.24892
	20.1–25.0	42	0.00029	0.01711	0.02164	-0.54060
	25.1–30.0	41	-0.00199	0.02640	0.03399	-0.90865
	30.1–35.0	35	-0.00482	0.04727	0.05906	1.02245
	>35.0	40	0.00097	0.05451	0.06978	-0.12242
Height class (m)	<15.0	20	-0.00155	0.00813	0.01057	-1.50575
	15.1–20.0	55	0.00334	0.01536	0.02297	1.06425
	20.1–25.0	95	-0.00393	0.03244	0.04330	-1.06558
	>25.0	30	0.00239	0.06360	0.07762	0.20716
Outside bark						
Diameter class (cm)	<15.0	14	-0.00045	0.00473	0.00530	-1.13484
	15.1–20.0	28	0.00429	0.00951	0.01247	1.91390
	20.1–25.0	42	-0.00150	0.01866	0.02454	-0.96007
	25.1–30.0	41	-0.00114	0.02872	0.03750	-0.74699
	30.1–35.0	35	-0.00339	0.04609	0.05843	0.74215
	>35.0	40	0.00228	0.06282	0.07915	-0.09566
Height class (m)	<15.0	20	-0.00208	0.00772	0.01062	-1.20203
	15.1–20.0	55	0.00243	0.01511	0.02290	0.82204
	20.1–25.0	95	-0.00344	0.03448	0.04598	-0.98562
	>25.0	30	0.00095	0.07195	0.08676	0.31376
Merchantable						
Diameter class (cm)	<15.0	14	-0.00045	0.00648	0.00800	-2.39296
	15.1–20.0	28	0.00441	0.01051	0.01509	2.00335
	20.1–25.0	42	0.00090	0.01662	0.02162	-0.54967
	25.1–30.0	41	-0.00200	0.02663	0.03402	1.00800
	30.1–35.0	35	-0.00556	0.04592	0.05821	-1.16844
	>35.0	40	0.00041	0.05263	0.06708	0.07295
Height class (m)	<15.0	20	-0.00181	0.00829	0.01180	-2.02629
	15.1–20.0	55	0.00336	0.01577	0.02323	1.02715
	20.1–25.0	95	-0.00425	0.03182	0.04272	-1.18392
	>25.0	30	0.00500	0.06104	0.07465	0.36232

Conclusion

Two mathematically consistent volume equations were evaluated for white spruce and white pine plantations in Ontario, Canada. Although the dimensionally compatible volume equation had better fit statistics (lowest MSE, AIC, and -2 log likelihood) and predictive ability than the combined variable volume equation, for white spruce the estimated values of the intercept were nonsignificant in the regression for total volume. The combined variable volume equation also provided logically consistent parameter estimates for these volumes for this species. Therefore, the combined variable volume equation was selected for estimating total volume for white spruce.

For merchantable volume, all estimated parameters were logically consistent in the dimensionally compatible volume equation. Since this equation also had better fit statistics and predictive accuracy, it was selected as the merchantable volume equation for white spruce plantations. For white pine, dimensionally compatible volume equations provided better fit and predictive ability for total and merchantable volumes and was selected for both volume equations for this species.

A nonlinear mixed effect modelling approach was used to fit equations for total and merchantable volumes for both species. The power function with base term DBH resulted in better fit than an exponential function and was used as the variance function for all volume equations.

The total volume equations (equations 9, 11) were logically consistent; i.e., even when DBH was zero, estimated total tree volumes were more than zero. Similarly, the estimated value of inside bark volume was smaller than outside bark volume for all trees.

References

Akaike, H. 1978. A Bayesian analysis of the minimum AIC procedure. Annals of the Institute of Statistical Mathematics 30: 9–14.

Avery, T.E. and H.E. Burkhart. 2001. Forest Measurements (5th edition). McGraw-Hill Education, New York, NY. 456 p.

Hayden, J., J. Kerley, D. Carr, T. Kenedi and J. Hallarn. 1995. Ontario Forest Growth and Yield Program field manual for establishing and measuring permanent sample plots. Ontario Ministry of Natural Resources, Ontario Forest Research Institute, Sault Ste. Marie, ON.

Honer, T.G. 1967. Standard volume tables and merchantable conversion factors for the commercial tree species of central and eastern Canada. Canadian Forest Service, Forest Management Research and Services Institute, Ottawa, ON. Information Report FMR-X-5. 21 p.

Pinheiro J.C. and D.M. Bates. 2000. Mixed-Effects Models in S and S-PLUS. Springer-Verlag. New York, NY. 528 p.

SAS Institute Inc. 2015. SAS/IML® 14.1 User's Guide. SAS Institute Inc. Cary, NC.

Schlesinger, W.H. 1991. Biogeochemistry, An Analysis of Global Change. Academic Press, New York, NY. 688 p.

Sharma, M. 2019. Inside and outside bark volume models for jack pine (*Pinus banksiana*) and black spruce (*Picea mariana*) plantations in Ontario, Canada. The Forestry Chronicle 95(1): 50–57.

Sharma, M. 2020. Increasing volumetric prediction accuracy—an essential prerequisite for end-product forecasting in red pine. Forests 11(1): 1050.

Sharma, M. 2021. Total and merchantable volume equations for 25 commercial tree species grown in Canada and the northeastern United States. Forests 12(9): 1270.

Sharma, M. and R.G. Oderwald. 2001. Dimensionally compatible volume and taper models. Canadian Journal of Forest Research 31(5): 797–803.

Sharma, M. and J. Parton. 2009. Modeling stand density effects on taper for jack pine and black spruce plantations using dimensional analysis. Forest Science 55(3): 268–282.

Spurr, S. 1952. Forest Inventory. The Ronald Press Company, New York, NY. 476 p.

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ISBN 978-1-4868-7396-8 (pdf)